Code No: 1076/C/R19

FACULTY OF SCIENCE

B.Sc. **(CBCS)** III-Year (VI-Semester) Regular & Backlog Examinations, June-2023

Mathematical Modeling (Optional)

Time: 3 Hours Max Marks: 80

SECTION-A (4x5=20 Marks)

Answer any Four questions from the following

1. Explain about Stochastic model.
2. Find an expression for the time for the population to double in size.
3. Explain about Newton's law of cooling.
4. Obtain a solution for differential equation with boundary conditions

   $$\frac{d^2U}{dx^2} - 4U = 0;\ U(0) = 0, U(1) = 3.$$
5. Check the Lotka-Volterra model in the limiting cases of prey with no predators, or predators with no prey.
6. Find an expression for the heat flux through the wall.

SECTION-B (4x15=60 Marks)

Answer the following questions

7. (a) Obtain a mathematical model for exponential decay and solve the IVP problem $\frac{dN}{dt} = -kN,\ \ N(0) = n_0$ on the interval $[0,1]$.

   (OR)

   (b) Find a differential equation for the amount of salt in the tank at any time t. (Note that concentration can be defined as the mass of salt per unit volume of mixture).

8. (a) Solve the logistic differential equation $\frac{dX}{dt} = rX(1 - \frac{X}{K})$ with the initial condition $X(0) = X_0$.

   (OR)

   (b) Determine a compartmental diagram and appropriate word equation for each of the two populations, the predator and the pray.

9. (a) Formulate a differential equation for the temperature of a cup of coffee over time.

   (OR)

   (b) Obtain a differential equation for the equilibrium temperature inside the annular shell.

10. (a) Explain and obtain a mathematical model for Lake pollution.

    (OR)

    (b) Solve the differential equation $\frac{d}{dr}\left(r\frac{dU}{dr}\right) = 0$ with boundary conditions $U(a) = u_a$; $J(b) = h(U(b) - u_b)$ and hence find an expression for the equilibrium temperature $U(x)$.